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Stochastic Frontier Production Function, Application and Hypothesis Testing

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ABSTRACT

In this study, the technical efficiency of the wheat farmers in the mixed farming system of the Punjab was estimated by using stochastic frontier production function, incorporating technical inefficiency effect model. The Cobb Douglas production function was found to be an adequate representation of the data, given the specification of the corresponding translog frontier model. The technical inefficiency effects were found present and contained a significant random element. The technical inefficiency effects were found to be a linear function of different firm specific factors. The individual impacts of some of the variables in the inefficiency effect model were non-significant, but the combined influence of all the ten variables was significant in reducing the inefficiency of the wheat farmers in the mixed farming system of the Punjab, Pakistan. The results also indicated that the farmers were operating at constant returns to scale.

Key Words: Stochastic; Frontier; Function; Hypothesis

INTRODUCTION

Wheat is an important food crop of the world. It was grown in 121 countries on 215 million hectares in 1999 representing 15 % of the world's cropped area. Over 583 million tons of wheat was harvested in 1999, which constituted 28 % of the world food supplies, of which 17 % was traded in the world market (FAO, 2000).

Global wheat consumption was up for the fifth year in a row and it exceeded wheat production. Consequently, wheat stocks tended to deplete, particularly among the importers (FAS, 2002). Developing world's share of all wheat imports amounted to 60% and had been increasing for 25 years and was expected to continue unabated (FAO, 2003). Out of the 121 countries growing wheat, Pakistan ranks 8^{th} in terms of area and production but 29^{th} in terms of yield per unit area (FAO, 2000).

In the recent years, country was self sufficient in wheat but fast and steady rising human population is very likely to upset food balance in the foreseeable future. For the year 2022, for example, wheat production has been projected at 23.71 million tons. On the other hand, wheat requirements have been projected at 28.92 million tons, showing a deficient of 5.2 million tons (Hammad, 1999).

Faster growth rate of human population is squeezing land and water availability on per capita basis. In 1972, for example, cultivated area averaged 0.72 acres which has been reduced to 0.36 acres in 2002-03 showing that in future more and more persons would live on less and less arable land (GOP, 1975; 2003). Similarly, surface water availability is deteriorating on per capita basis. In 1951, about 5300 cubic meters water was available on per capita basis; this has gone down to 1200 cubic meters in 2000. Despite the fact that Pakistan possesses the worlds' largest surface irrigation net work, yet the per capita surface water availability is falling drastically over time (GOP, 1955; 2003).

Another disquieting factor in food sufficiency is rapidly deteriorating status of ground water. At present about 40% of irrigation requirements are met by the ground water. Studies show that water being pumped out by 25% of tubewells is of marginal quality, while another 50 % tubewells pump out hazardous water, this adds significantly towards salinization of the productive arable land (Malik *et al*., 1991). On this reason about 100 million tons of salts are added to soil every year (ICID, 1991). About 50 million tons of salt are being added to the system every year by canal irrigation system (Qureshi, 1993). Consequently, productivity has fallen over time for almost all the major crops to the extent of 25-70%. In sever cases it goes up to 100%. At the present stage of development, especially in the face of explosive population growth, Pakistan can ill afford to see its crop productivities declining due to low crop germination rates, poor and uneven establishments of crops in the field.

The study in hand is oriented towards the goal of achieving higher productivity by improving technical efficiency of the wheat farmers. Main objectives of the study are as follows:

OBJECTIVES

1. Whether technical efficiency effects are present or not.
2. If present, do they contain a significant random element or not.
3. Technical inefficiency effects are influenced by farm specific factors or not.

4. To check the return to scale in wheat farming in the mixed farming system of the Punjab.

Stochastic frontiers and efficiency measurement. The measurement of the efficiency of production has been an important area of research over the last two decades. For this purpose stochastic frontier production function has been used. Coelli (1996) observed that thirty out of forty studies on application of frontier models to agriculture have used stochastic frontier production function. The advantage of using stochastic frontier models are: (1) It introduces a disturbance term representing statistical noise, measurement error and exogenous shocks beyond the control of production units which would other-wise be attributed to technical inefficiency, (2) It provides the basis for conducting statistical tests of hypothesis regarding the production structure and the degree of inefficiency. The estimation of frontier function and efficiency can be completed either in one stage or in two stages. Parikh and Shah (1996) presented a review of the various approaches to efficiency measurement and conducted empirical analyses of cross-sectional data from 397 sample farmers in the North-West Frontier Province of Pakistan. In their stochastic frontier analysis, a two-stage approach was used. In the first-stage analysis, a Cobb-Douglas stochastic frontier production function was estimated. The total value of agricultural output per acre was modeled in terms of five input variables, namely, cost of manure, cost of fertilizers, wages for human labour, cost of animal labour and tractor costs (all on a per acre basis). The technical efficiencies of production were also estimated using the approach of Jondrow, *et al*. (1982). In the second-stage of the analysis of Parikh and Shah (1996), the estimated technical efficiencies were regressed on various farm-and farmer-specific variables, which were considered appropriate in explaining variations in technical efficiencies for the sample farmers.

The two-stage analysis of explaining levels of technical efficiency (or inefficiency) was criticized by Battese and Coelli (1995) as being contradictory, in the assumptions made in the separate stages of the analysis. In this paper, we follow the Battese and Coelli (1995) approach of modeling both the stochastic and the technical inefficiency effects in the frontier, in terms of observable variables, and estimating all parameters by the method of maximum likelihood, in a single-step analysis.

Model and variables. The study used the primary data which were collected from 112 wheat farmers located on the head, middle and tale of the lined/unlined water courses in the mixed farming system. The Cobb-Douglas (CD) production function was found to be an adequate representation of the data, given the specifications of the corresponding translog frontier model. The stochastic frontier model is defined by:[1]

Where ln represents the natural logarithm (base, e); the subscript, i denotes the i-th farmer in the sample, i=1,2,……..112;

Wheat production, (Y_i) represents the total wheat production (in maunds i.e. 40 kgs) for the farmer; Wheat area (X_{1i}) represents the total area of wheat (in acres); Irrigation (X_{2i}) represents the quantity of irrigation water applied to the wheat crop, which is defined as the number of irrigations times the area of wheat grown; Weedicide (X_{3i}) represents the total cost of weedicide applied to the wheat crop, which is defined as cost of weedicide per acre times area of wheat grown; Cultivation (X_{4i}) represents the total number of cultivations given to the wheat crop, which is defined as number of cultivations per acre times area of wheat grown; Fertilizer (X_{5i}) represents the total nutrient kgs of fertilizer applied to the wheat crop, which is defined as nutrient kg of fertilizer per acre times area of wheat grown; Farm Yard Manure (X_{6i}) represents the total number of trollies applied to the wheat crop, which is defined as the number of trollies applied per acre times area of wheat grown; Family Labour (X_{7i}) represents the total number of adult male equivalents available on the farm and Seed (X_{8i}) represents the total quantity of seed sown, which is defined as the quantity of seed used per acre times area of wheat grown, the β_ks, k = 0, 1, 2, 3, 4, 5, 6, 7, 8, are unknown parameters for the production function; the V_is are random errors associated with measurement errors in the production of wheat reported, or the combined effects of input variables not included in the production function, where as V_is are assumed to be independently and identically distributed N(0, σ^2_V) random variables; the U_is are non-negative random variables, associated with technical inefficiency of production of the farmers, assumed to be independently distributed, such that the technical inefficiency effect for the i-th farmer, U_i, is obtained by truncation (at zero) of the normal distribution with mean U_i, and variance, σ^2, such that[2]

Where, Z_{1i} represents the operational farm area in acres; Z_{2i} is a dummy variable for sowing time (if the wheat crop is sown in time, then it has a value of one, otherwise zero); Z_{3i} represents the age of farmers in years; Z_{4i} represents the education of farmers in years of schooling; Z_{5i} is a dummy variable indicating the location of farm on the watercourse (if the farm is located at the head of the watercourse, then it has a value of one, otherwise zero); Z_{6i} is a dummy variable indicating the watercourse (if the watercourse is lined, then it has a value of one, otherwise zero); Z_{7i} represents the canal water shortage measured as the %age of total water used supplied by the tubewell; Z_{8i} is a dummy variable for credit (if the farmer acquired credit, then it has a value of

[1] $\ln(Y_i) = \beta_0 + \beta_1 \ln(X_{1i}) + \beta_2 \ln(X_{2i}) + \beta_3 \ln(X_{3i}) + \beta_4 \ln(X_{4i}) + \beta_5 \ln(X_{5i}) + \beta_6 \ln(X_{6i}) + \beta_7 \ln(X_{7i}) + \beta_8 \ln(X_{8i}) + V_i + U_i$

[2] $U_i = \delta_0 + \delta_1 Z_{1i} + \delta_2 Z_{2i} + \delta_3 Z_{3i} + \delta_4 Z_{4i} + \delta_5 Z_{5i} + \delta_6 Z_{6i} + \delta_7 Z_{7i} + \delta_8 Z_{8i} + \delta_9 Z_{9i} + \delta_{10} Z_{10i}$

one, otherwise zero); Z_{9i} is a dummy variable for sowing method (if the farmer had sown their crop with drill, then it has a value of one, otherwise zero); Z_{10i} is a dummy variable for tubewell (if the farmer had his own tubewell, then it has a value of one, otherwise zero) and the δ_s are unknown parameters to be estimated.

This stochastic frontier model is estimated using the computer program, FRONTIER 4.1, written by Coelli (1996). The parameters of the frontier model are estimated, such that the variance parameters are:

$$\sigma_S^2 = \sigma_V^2 + \sigma^2 \quad \text{and} \quad \gamma = \sigma^2 / \sigma_S^2$$

where the γ parameter has a value between zero and one.

RESULTS

The maximum likelihood estimates of the parameters of the stochastic frontier production function defined by equation 1 and 2 are presented in Table I along with their standard errors and t-values.

The Cobb Douglas production function was found to be an adequate representation of the data, given the specification of the corresponding Translog Frontier Model. The values of the logarithm of the likelihood function for the Cobb Douglas and Translog Frontier Model were 63.44 and 81.305, respectively. As a result the generalized likelihood ratio test statistics came out to be 35.73, which is less than the critical chai square table value of 43.77 at 36 degree of freedom (the difference between the numbers of parameters of the two models) and 5% level of significance. This suggested that Cobb Douglas Frontier Model was an adequate representation of the data. As Cobb Douglas production function is used in model defined by equation 1, so these β-estimates are the average elasticities of production.

The estimate for the variance parameter, $\sigma^2/\sigma^2 s$, indicates that the variance, σ^2, associated with the inefficacy effect is about 60% of the two variances.

Various restrictions were imposed on the model defined by equation 1 and 2. To check whether these restrictions were valid or not, the generalized likelihood ratio tests were used. The results of these tests of hypothesis for parameters of the stochastic frontier and inefficiency effects model for wheat farmers in Toba Tek Singh district are presented in Table II. The first column of the Table II represents the restriction imposed or the null hypotheses.

Table I. Maximum likelihood Estimates for Parameters of Stochastic Frontier Production Function and Inefficiency Model for Wheat Farmers in the mixed farming system of Punjab, Pakistan

Variable	Parameter	Standard error	T-Value
β_0	3.558	0.596	5.97
Ln of Wheat area (acres)	0.783	0.183	4.28
Ln Irrigation (No.)	0.046	0.005	0.86
Ln Weedicide cost (Rs.)	0.029	0.012	2.51
Ln Cultivation (No.)	0.323	0.052	6.18
Ln Fertilizer (N.kg)	0.201	0.474	4.24
Ln FYM (Trollies)	0.007	0.008	0.89
Ln Family labour	0.041	0.033	1.24
Ln Seed (Kg.)	-0.395	0.162	-2.45
Inefficiency Model			
δ_0	-0.135	0.715	-0.19
Farm area (acres)	-0.001	0.008	-0.14
Sowing time (dummy)	-0.088	0.081	-1.09
Age (years)	-0.003	0.005	-0.67
Education (years)	-0.031	0.015	-2.03
Location of farm (dummy)	-0.0009	0.097	-0.009
Water course (dummy)	-0.097	0.163	0.59
Water shortage (%age)	0.010	0.006	1.61
Credit (dummy)	-0.311	0.214	-1.45
Drill	-0.389	0.003	-1.29
Owned Tubewell (dummy)	-0.070	0.100	-0.69
Variance Parameters			
σ_s^2	0.039	0.007	5.18
γ	0.594	0.136	4.36
Log-likelihood Function	63.44		

The second column shows the value of the log likelihood statistics when the restriction present in the first column is imposed on the original model. The third column represents the calculated test statistic. The fourth column represents the critical values for the test statistic present in column three. The fifth column represents the decision i.e. whether restriction is valid or not or null hypothesis is accepted or rejected. In Table II, the first null hypothesis tested is that technical inefficacy effects are absent from the model. The omission of U_i is equivalent to imposing the restriction specified in the null hypotheses i.e.

Ho : $\gamma = \delta_0 = \delta_1 = \delta_2 = \delta_3 = \delta_4 = \ldots\ldots\ldots\ldots = \delta_{10} = 0$

When this restriction was imposed on the model represented by equation 1 and 2, the value of the logarithm of the likelihood function (LLF) reduces to 50.02. This provides generalized likelihood ratio (LLR) test statistic of 26.84, which is larger than the critical value range of 5.14 to 13.40. Thus we reject the null hypothesis of no technical inefficiency effects, given the specifications of the stochastic frontier and inefficiency effect model. The second

Table II. Tests of Hypothesis for Parameters of the Stochastic Frontier and Inefficiency Model for Wheat Farmers in the mixed farming system of Punjab, Pakistan

Null Hypotheses	Log likelihood statistics	Test statistics	Critical value	Decision
$H_0 : \gamma = \delta_0 = \delta_1 = \ldots = \delta_{10} = 0$	50.02	26.84	5.14-13.40	H_0 Rejected
$H_0 : \gamma = 0$	58.96	8.96	5.14-7.05	H_0 Rejected
$H_0 : \delta_0 = \delta_1 = \ldots = \delta_{10} = 0$	50.64	25.60	19.68	H_0 Rejected
$H_0 : \beta_1 + \beta_2 + \ldots + \beta_8 = 1$	62.78	1.32	3.84	H_0 Accepted

null hypothesis or resection considered in Table II is Ho: $\gamma = 0$, which specifies that technical inefficiency effects are not stochastic. If the parameter γ is zero, then the variance of the technical inefficiency effect is zero and so the model reduces to the traditional mean response function in which the sowing time, education, age, water course position, farm area, water course (lined/unlined), water shortage, loan, sowing method and tubewell (owned) variables are included in the production function. However, if the γ parameter is equal to zero, then the δ_0 parameter is not identified, given that the production function has an intercept. When this restriction was imposed on the model, the value of the logarithm of the likelihood fraction reduces to 58.96. This provides a generalized likelihood ratio test statistic of 8.96, which is larger then the critical value range of 5.14 to 7.05. Thus, the null hypothesis that the technical inefficiency effects are not random is rejected.

Another question of particular interest to this study is whether the ten firm-specific factors, considered in the inefficiency model, have a significant influence upon the degree of technical inefficiency associated with the wheat farmers. Thus a test of null hypothesis that,

Ho:$\delta_0 = \delta_1 = \delta_2 = \delta_3 = \delta_4 = \ldots\ldots\ldots\ldots\ldots = \delta_{10} = 0$ is conducted. When this restriction was imposed on the model, the value of the logarithm of the likelihood fraction reduces to 50.63. This provides a likelihood ratio test statistic of 25.6, which is larger than the critical value of 19.68. Thus the null hypothesis that ten firm specific factors do not have an influence upon the technical inefficiency is also rejected. This indicates that the joint effect of these ten explanatory variables on the levels of technical inefficiencies is significant, although the individual effects of some of the variables are not statistically significant.

The last hypothesis in the Table II is that there are constant returns to scale in the wheat farming. Thus a test of null hypothesis that

Ho: $=\beta_0 = \beta_1 = \beta_2 = \beta_3 = \beta_4 \ldots = \beta_8 = 1$ is conducted.

When this restriction was imposed on the model, the value of the logarithm of the likelihood function becomes 62.78. This provides the likelihood ratio test statistic of 1.32, which is less then the critical value of 3.84. Thus, the null hypothesis that there is constant return to scale is not rejected. Thus the return to scale parameter 1.03 is not significantly different from one.

CONCLUSIONS

On the basis of empirical results we come up with the following conclusions.

1. Technical inefficiency effects are present and contain a significant random component. Except for few years, Pakistan has been a net wheat importer. In order to meet the food requirement of rapidly increasing population, to save foreign exchange and for food security, we must increase wheat production by utilizing all possible means. By reducing technical inefficiency we can increase on an average wheat production by 6.4% in the mixed farming system of the Punjab.
2. The joint effect of the ten explanatory variables included in the inefficiency effect model is significant in reducing the technical inefficiency of wheat farmers, although the individual impact of some of these variables were non-significant.
3. There are constant returns to scale in wheat farming in the mixed farming system of the Punjab. So there is no scope for increasing wheat production by increasing all the inputs in the same proportion.

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